The Algebra Problem

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I. Overview

There is general agreement in our state and in our nation that mathematics achievement must improve significantly in order for us to compete successfully in the global, knowledge-based economy of the 21st century. Too many of our high school graduates are not mathematically prepared for postsecondary education or work. Too few are mathematically prepared to pursue a college degree in vitally important Science, Technology, Engineering and Mathematics (STEM) disciplines.

The nature and extent of the mathematics achievement problem, called The Algebra Problem in this paper, are discussed below. The paper builds upon discussions at a mathematics forum held at Dixie Heights High School last November. It was my privilege to be one of four speakers at this forum, co-sponsored by the Kenton County School District and the Northern Kentucky Council of Partners.

Discussions with the audience that followed the presentations seemed to resonate with everyone involved. These discussions focused on the idea that Kentucky requires its teachers to cover too many topics in too little depth in order to prepare students for state assessment tests. As a result, too many students fail to master essential topics necessary for success at the next learning level, be it at middle school, high school, postsecondary education or work. Too many incur learning deficiencies that impede their academic progress and that are difficult to remediate in later years.

This is a national problem that is certainly not unique to Kentucky. In 2006, the National Council of Teachers of Mathematics (NCTM) publication, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*, expressed this problem concisely in its preface:

As state and local school districts implement more rigorous assessment and accountability systems, teachers often face long lists of mathematics topics or learning expectations to address at each grade level, with many topics repeating from year to year. Lacking clear, consistent priorities and focus, teachers stretch to find the time to present important mathematical topics effectively and in depth.

A clear understanding of the problem leads naturally to some possible solutions. Foremost among them is to implement the suggestions made by the NCTM in its focal points publication. Another is to build on the extensive collaborative efforts of the Kentucky postsecondary and secondary mathematics communities that have been facilitated by the Council on Postsecondary Education and the Kentucky Department of Education. These efforts have helped to clarify the essential mathematics skills and concepts students need to be successful. These clarifications can and should be used to help focus the high school mathematics curriculum in the same way as the NCTM focal points help focus the K-8 mathematics curriculum.

It is my hope that this paper will generate extensive discussions about how we can collectively and collaboratively solve The Algebra Problem.

II. Nature and Extent of The Algebra Problem

1. Remediation Rates. The unacceptably large proportion of incoming college students who are under-prepared in mathematics is the most visible manifestation of The Algebra Problem.

Table 1 gives the public college remediation rates in mathematics compiled by the Council on Postsecondary Education (CPE). It shows that 41% of the incoming public college students were under-prepared in 2002 and 44% were under-prepared in 2004. Students are currently considered under-prepared in mathematics if their math ACT score (or equivalent score on the COMPASS test) is less than 18.

In the fall of 2009, this math ACT score will go up from 18 to 19. It is reasonable to assume that this change will push the remediation rate in mathematics to over 50%. This assumption is based on 2004 data showing that the percentage of incoming Kentucky public college students with a math ACT score less than 18 was 42.5 whereas the percentage with a math ACT score less than 19 was 50.7.

Table 1
Public College Remediation Rates in Mathematics

Year	Percentage Under-Prepared	Math ACT Score
2002	41	<18
2004	44	<18
2009 (Estimated)	>50	<19

2. Developmental Mathematics. It is not commonly understood that essentially all developmental (the term generally used to replace "remedial") mathematics courses, nationwide and in Kentucky, are in either algebra or pre-algebra. There are very few developmental courses in geometry and there are no developmental courses in data analysis, probability and statistics.

The reason there are no developmental courses in these three areas is that students can learn what they need to know about these areas, without remediation, in entry-level, college mathematics courses provided they have reasonably good algebra, arithmetic and reading skills.

3. Failure Rates in Developmental Mathematics. Under-prepared students placed into developmental mathematics courses face an uphill battle in trying to complete successfully the one or more developmental mathematics courses they must take. The learning deficiencies they have incurred in algebra and arithmetic make these courses very difficult.

This is true nationally as well as in Kentucky. The National Center for Educational Statistics reports that the college courses with the highest failure rates (the percentage of

students who enroll in a course and who either fail it or withdraw from it) are developmental elementary algebra (roughly high school Algebra I), developmental intermediate algebra (roughly high school Algebra II) and developmental pre-algebra (roughly middle school mathematics).

Data compiled by the CPE show that the situation in Kentucky is no different (http://cpe.ky.gov/info/dev_edu). The report, *The Developmental Education of Kentucky's Entering Postsecondary Class of 2004*, shows that 48% of Kentucky public college students whose first college mathematics course was a developmental mathematics course did *not* successfully complete this course with a grade of "D" or better. In other words, 48% of these students either failed the course or withdrew from it.

Institutional data at Northern Kentucky University (NKU) aligns well with what the National Center for Educational Statistics reports nationally. NKU examined 190 courses taken by its freshman from 2000 to 2005, and compiled the number and percentage of these students who either failed the course (a grade of "F") or withdrew from it (a grade of "W"). Partial results are shown in Table 2. A low-enrollment philosophy class had the highest failure rate, but the next four were all algebra or pre-algebra classes.

Table 2
NKU Freshman Failure Rates (190 courses from 2000 through 2005 examined)

Rank	Course	Number of Students	Grade of F or W
2.	Developmental Elementary Algebra	6405	41.8%
3.	Developmental Pre-Algebra	560	41.6%
4.	College Algebra	1457	40.5%
5.	Developmental Intermediate Algebra	4341	39.3%

4. College Algebra and Calculus. The Algebra Problem is just as serious in college-level mathematics and science classes as it is in developmental mathematics. This is particularly true in algebra-intensive courses such as college algebra and calculus.

College algebra is an entry-level mathematics course offered at every public college and university in Kentucky and in almost all colleges and universities nationwide. It is a required course for a wide variety of college majors in the sense that students must either take and pass the course, or place out of it, in order to complete their major. It is required for a major in any Science, Technology, Engineering and Mathematics (STEM) discipline and for a major in any business discipline at any public university in Kentucky.

It goes without saying that a good background in high school algebra is essential for success in college algebra. Yet too many students lack the essential arithmetic and high school algebra skills to be successful in college algebra. Thus the failure rate statewide in this course is depressingly high and likely somewhere around 40%, just as it is at NKU (see Table 2).

Calculus is required for a major in most STEM disciplines. Every calculus instructor in the state and in the nation will tell you that a good background in algebra is *by far* the most important prerequisite for calculus. Students have a good chance of being successful in calculus provided they have a solid background in algebra; otherwise, they have little chance for success.

Students all too often come to college intending to major in a discipline that will lead to a high-paying and prestigious career only to find that they do not have the mathematics background to complete this major. They are forced to change majors and often drop out of college altogether as a result. This is a tragedy for them and for a state that needs the intellectual capital and expertise that they could supply.

5. STEM Majors. There is a growing realization that the United States must produce more college graduates in the STEM disciplines in order to remain competitive in the global economy. Our country must make dramatic changes soon or risk becoming a second-rate nation.

Our education system must take the lead. Our prospects, based on international comparisons, do not look good. We are consistently being out-performed in mathematics and science by other industrialized nations. The Programme for International Student Assessment (PISA) results in science and mathematics are characterized as *"scary!"* (http://mwhodges.home.att.net/new_96_report.htm). In 2006, for example, U. S. 15-year-old students ranked 35th out of 57 nations in mathematics, behind every other industrialized nation except Italy. In 2003, this same age group ranked 29th out of 34 nations.

Kentucky has joined the nation in its concern about educational attainment in mathematics and science. A statewide STEM Task Force was formed and many excellent recommendations were made in its final report. The report can be found online at (http://cpe.ky.gov/committees/stem/default.htm).

How is Kentucky doing in preparing its students for careers in the STEM disciplines? The answer is not encouraging. Table 3 shows the percentage of Kentucky students at or above the ACT benchmark score on each of the four ACT sub-tests in 2007. The benchmark score in mathematics is 22. What this means is that students who score at or above 22 on the math ACT test have a good chance of earning a grade of "C" or better in college algebra, a conclusion based on extensive ACT research. The benchmark scores in science, reading and English are determined in a similar manner.

The data in Table 3 show that Kentucky is significantly behind the nation in mathematics, with only 35% of our students prepared for college algebra as opposed to 43% nationwide. Since a strong background in mathematics is critically important for students who intend to major in a STEM discipline, this glaring deficiency is not a good sign. Kentucky is lagging well behind a nation that is lagging well behind the industrialized world!

What is particularly discouraging is that precisely the same cohort of students performed so much worse in mathematics than in the other three ACT-tested areas. Unfortunately, this characteristically poor performance in mathematics as compared with the other three areas has occurred consistently for at least a decade.

Table 3
2007 ACT Data

ACT Sub-Test	ACT Benchmark Score	National % at or above Benchmark	Kentucky % at or above Benchmark	Kentucky % behind the Nation
Mathematics	22	43	35	8
Science	24	28	24	4
Reading	21	53	50	3
English	18	69	67	2

5. Roots of the Algebra Problem. Efforts to solve The Algebra Problem naturally focus on the high school mathematics curriculum. But it is not that simple. Because mathematics is a cumulative discipline where new content is built upon previous content throughout the K-12 curriculum and beyond, the entire K-12 curriculum must be examined if The Algebra Problem is to be solved.

Arithmetic is just as serious a concern as algebra and can be viewed as the root of The Algebra Problem. Students must learn essential arithmetic skills before they can understand algebra just as students must learn essential algebra skills before they can understand calculus.

There is overwhelming anecdotal evidence that students are taking algebra without having learned the essential arithmetic skills and concepts necessary to be successful. The failure rate is greater in Algebra I than in any other high school course just as the failure rate is greater in its college equivalent, developmental elementary algebra, than in any other college course.

Algebra teachers at both the secondary and postsecondary levels are increasingly alarmed that too may of their students do not know their multiplication tables, cannot add, multiply and divide fractions and decimals, cannot estimate answers to arithmetic calculations or determine whether their answers are reasonable. In short, these students have neither computational fluency in, nor understanding of, arithmetic.

These students are often totally dependent on their calculators to do even the simplest arithmetic. My experience last semester in a college algebra class is but one illustration of the problem. I gave the class an algebra problem that required them to multiply 1/3 by 3. Most of them reached for their calculator because they were insecure doing this calculation without one. A few did not have a calculator, and so they cancelled the 3's, got 1/1, but did not realize that 1/1 is equal to 1.

It is very difficult to teach algebra to students so deficient in arithmetic. Any algebra teacher, be it at the secondary or postsecondary level, will confirm this observation. It is even more difficult to remediate these deficiencies in arithmetic at the secondary and postsecondary level. The best time to learn arithmetic is in the elementary grades at the developmentally appropriate age.

7. Proficiency and Preparedness. The Kentucky Board of Education (KBE) is currently considering the meaning of proficiency and trying to determine whether proficiency and preparedness mean the same thing.

There is good reason to do so. Scores on the Commonwealth Accountability and Testing System (CATS) tests keep rising at a steady rate and yet college remediation rates remain unacceptably high with no sign of improvement. In fact, with both college attendance and college-readiness standards increasing, it appears that these remediation rates will *increase significantly* in coming years.

Perhaps an even better reason to question whether proficiency means preparedness is to look at CATS scores in mathematics and science at the elementary, middle, and high school levels as projected for 2014. These projections were discussed at the June 2007 KBE meeting and are shown in Table 4.

Table 4
CATS Scores Projected for 2014 in Math and Science

	Elementary School	**Middle School**	**High School**
Mathematics	105.4	85.0	84.0
Science	110.7	93.0	81.6

Elementary schools statewide are projected to be well above the proficiency level of 100 in both mathematics and science, whereas middle schools and high schools are projected to be well below the proficiency level of 100 in these disciplines. In mathematics, there is a steep decline in scores of over 20 points from the elementary to the middle school level.

Ideally, if students are proficient in mathematics and science coming out of elementary school, then they should be prepared to succeed in these disciplines in middle school. But the data suggest that this may not be the case. It appears that a significant proportion of students who are proficient coming out of elementary school might not be prepared for middle school. In some Northern Kentucky school districts, for example, close to 80% of the students are at the proficient level or above on the elementary school CATS test in mathematics, and yet middle school and high school mathematics teachers in these same districts indicate that a great many of their students do not know their multiplication tables and cannot add, multiply and divide fractions and decimals.

It is critically important that the testing system be aligned so that students proficient at one level are prepared for success at the next level.

III. Recommendations for K-8 Mathematics

1. NCTM Focal Points. One very promising way to solve The Algebra Problem at its inception in grades K-8 is to adopt the suggestions made by the National Council of Teachers of Mathematics (NCTM) in its landmark publication, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (http://my.nctm.org/ebusiness/ProductCatalog/product.aspx?ID=13089).

The focal points consist of only three major learning objectives at each grade level. The idea is to provide coherence in the curriculum by focusing in depth on a small number of essential mathematical topics at each grade level, and hence move away from the "inch-deep, mile-wide" curriculum so prevalent in this country. Stated below are a few selected focal points that give the flavor of what is intended.

Grade 4

- *Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication.*
- *Developing an understanding of decimals, including the connections between fractions and decimals.*

Grade 5

- *Developing an understanding of and fluency with division of whole numbers.*
- *Developing an understanding of and fluency with addition and subtraction of fractions and decimals.*

Grade 6

- *Developing an understanding of and fluency with multiplication and division of fractions and decimals.*

The emphasis throughout is on fluency and understanding. Students who do not understand the concepts on which computational procedures are based are doomed to confuse and to forget them.

2. Rationale for Focal Points. The rationale for focusing the mathematics curriculum in a quest for coherence comes from the international and national K-12 mathematics communities, the postsecondary mathematics community and the business community.

The Third International Mathematics and Science Study (TIMSS [1997]; now known as the Trends in Mathematics and Science Study) analyzed the mathematics curricula in participating countries. The curriculum in other countries, nearly all of which out-performed the United States in mathematics and science, covered far fewer topics in much greater depth. This study led to the familiar characterization of school mathematics in the United States as "a mile wide and an inch deep."

The postsecondary mathematics community nationwide has long been concerned that incoming students lack the essential mathematics skills to be successful in college and has called for more focus on these skills in the school mathematics curriculum. The ACT

report, *Rigor at Risk* (http://www.act.org/path/policy/pdf/rigor_summary.pdf), summarizes this concern.

High school teachers and postsecondary faculty disagree about the depth and breath of essential state standards needed to prepare students for college. High school teachers rate a much larger number of topics and skills as being "important" or "very important" for college success than do college instructors. This parallels the tendency of many state standards to be broad and inclusive rather than specific and selective. It may be that the nature of state standards forces teachers to treat all topics as important, potentially sacrificing depth for breadth. In contrast, postsecondary educators indicate that a more rigorous treatment of fundamental *content knowledge and skills would better prepare students for college and work.*

The underlined portion above more accurately reflects the opinions of the high school mathematics teachers I have known. Indeed, these teachers are enthusiastic about working with their postsecondary counterparts in collaborative efforts to prepare students for postsecondary education.

The business community is concerned because their new employees often lack the mathematics skills to do their jobs effectively, frequently requiring some form of remediation. A common concern is the lack of essential arithmetic skills of the kind heavily emphasized in the focal points.

In response to these and other concerns, the National Council of Teachers of Mathematics developed the focal points.

3. Implementation of the Focal Points. Implementation of the focal points in any state should be relatively easy once it is understood that the focal points are not intended to replace state standards; rather, they are intended to focus these standards by identifying three important learning objectives at each grade level.

The NCTM focal points document is in fact careful to connect the focal points at each grade level to its primary and very extensive curriculum document, *Principles and Standards of School Mathematics*.

Kentucky or any other state could implement the focal points by relating them to state standards in the same way that the NCTM connected the focal points to its standards. There is no reason to revise state standards and this would make it much easier for states to implement the focal points.

4. Suggested Revision of CATS Test. There is one change Kentucky could make to align the CATS test with the focal points. That change would be to require students to complete a portion of the elementary and middle school CATS tests in mathematics without a calculator. The focal points put strong emphasis on computational fluency, and this clearly means that students should be able to do simple calculations without a calculator.

Teachers do in fact require students to do arithmetic without a calculator. But in a rush to cover required content, it is all too easy for teachers to permit students to use a calculator exclusively once they have been exposed to doing arithmetic without one. In too many cases, this leads to serious calculator dependency and a resulting lack of computational fluency. This is exactly the kind of problem the focal points are intended to correct.

Requiring students to complete a portion of the CATS test without a calculator will make it much more likely that teachers will emphasize computational fluency. This change will initially be distinctly unpopular with students. But in the long run, it will be of great benefit in preparing them for the future. It may also help close the gap between proficiency and preparedness so that students who are proficient coming out of elementary school are more likely to be prepared for middle school.

IV. Recommendations for High School Mathematics.

1. Need to Focus the High School Curriculum. Despite impressive progress made in aligning the high school mathematics curriculum with postsecondary expectations, there remains a need to focus the high school curriculum on a smaller number of fundamental skills and concepts that can be covered in greater depth and are essential for success in postsecondary education and work. This focusing of the curriculum is what postsecondary and business leaders are advocating, it is what is done internationally in K-12 systems, and it is what NCTM is advocated in this country.

A task force on postsecondary education convened by the Kentucky Chamber of Commerce released its report in December (http://www.kychamber.com). The report cites lack of alignment as a barrier to postsecondary education and reinforces the need to align state assessment tests with postsecondary expectations.

Although progress has been made, appropriate connections-also called alignment-do not exist between and among all levels of education to ensure the success of students. A striking example is the misalignment of the state assessment for high school students, the Commonwealth Accountability and Testing System or CATS, with the expectations for postsecondary-level study.

Kentucky is indeed fortunate to have a business community so willing and able to get actively involved in education.

2. Develop Focal Points for High School Mathematics. The need to focus the high school curriculum leads naturally to the idea of developing focal points along the lines of those developed by the NCTM for the earlier grades. At the high school level, however, it seems logical to develop focal points for specific courses such as Algebra I, Algebra II and Geometry rather than for each grade level.

The NCTM is considering the possibility of developing focal points for high school mathematics, but Kentucky should nevertheless move ahead with its own focal points as

soon as possible. Much work on curriculum alignment has already been completed, and it is important that secondary and postsecondary mathematics teachers begin building on the existing work.

Following the lead of the NCTM, the idea would be to select a small number of essential learning objectives for high school mathematics courses beginning with Algebra I, Algebra II and Geometry, and then connect these objectives to state mathematics standards. There would be no need to revise the standards.

3. Progress in Curriculum Alignment. Kentucky has made extraordinary progress in aligning the mathematics curriculum over the last six years. Many states and national education organizations are amazed by what has been accomplished and are most interested in how it was done. A few of the important accomplishments are discussed below.

Kentucky was one of five original partner states chosen to participate in the American Diploma Project (ADP) in 2002. After two years of work, the landmark report, *Ready or Not: Creating a High School Diploma That Counts* (http://www.achieve.org/node/552), was released in early 2004. The report contains benchmarks in mathematics and English language arts that states can use to align their standards.

In 2005, Achieve, Inc., the national educational organization that took the lead in the ADP project, launched the ADP Network consisting of states committed to improving their educational systems and aligning their standards in mathematics and English with the ADP benchmarks. Kentucky and 29 other states, educating over half the nation's students, are currently members of the ADP Network.

Kentucky vigorously followed up on the ADP work. The Council on Postsecondary Education (CPE) convened groups of mathematics and English faculty members in 2004, each with one representative from each state university and one from the community college system. The groups developed college readiness standards in mathematics and English based on the ADP benchmarks as well as a postsecondary placement policy in each discipline (http://cpe.ky.gov/committees/p16/).

In mathematics, the college readiness standards delineate the essential mathematics skills that students need to be successful in college. The postsecondary mathematics faculty members who developed the standards were in strong agreement about these essential skills, making construction of the standards a relatively easy task.

The Kentucky Department of Education (KDE) shortly thereafter convened the Commissioner's Task Force on Mathematics, a remarkably large and diverse group of secondary and postsecondary mathematics teachers, and representatives from business and education. The task force was charged with revising the core content for high school mathematics and aligning it with postsecondary expectations and the ADP benchmarks. There was a shared realization that Kentucky had a serious problem in mathematics and the task force members worked diligently together to write standards that would help

solve the problem. As a result, Kentucky now has dramatically improved state standards in high school mathematics.

The Statewide Mathematics Placement Testing Project is a recent initiative and is a work in progress. The idea is to develop a common set of online placement tests that can be used to reinforce and refine placement decisions in mathematics based on the ACT, and thereby enhance student success. The project is run by a loosely structured group consisting of math faculty from all eight state universities, including six department chairs; math faculty from 11 of the 16 community and technical colleges; math faculty from three private colleges; and representatives from the CPE and adult education.

A great deal of progress has been made since the project began in the spring of 2006. A developmental mathematics placement test, a college algebra placement test and a calculus placement test have been developed, discussed, modified, approved, and put online. The tests are offered *free of charge* to any educational institution in the state. Several colleges and universities have made extensive use of the secure, online testing system since it became available in the spring of 2007.

Perhaps the most important outcome of the project is the exceptionally strong postsecondary consensus that emerged about the essential algebra and pre-algebra skills students need to be successful in developmental mathematics courses and algebra-based courses such as college algebra and calculus. It should be emphasized that these skills are *necessary*, but not *sufficient*, for success in these courses. No one is advocating building a high school mathematics curriculum around these skills alone. On the other hand, high school students should fully understand how important these skills are for success in postsecondary mathematics and science courses.

In sum, Kentucky has done an extraordinary job of identifying the essential mathematical skills and concepts that students need to be successful in postsecondary education. The next step is to build upon these efforts in order to provide greater focus and coherence in the high school mathematics curriculum.

V. Conclusion

If we as a state are going to

- develop a robust economy that will permit us to compete in the global economy;
- significantly increase the number of Science, Technology, Engineering and Mathematics (STEM) majors we graduate;
- reduce remediation rates;
- double the number of college graduates by 2020;

then we must solve The Algebra Problem!

The postsecondary mathematics community in the state, the postsecondary mathematics community in the nation, the international K-12 mathematics community, the national K-

12 mathematics community, and the business community have reached a consensus about how The Algebra Problem can be solved: focus the mathematics curriculum by identifying the skills and concepts essential for student success, and then cover these skills and concepts in depth to insure that students master them.

Kentucky would be wise to build solutions to The Algebra Problem around this consensus. Our future depends upon it.

About the Author

Steve Newman is professor of mathematics at Northern Kentucky University and director of the Kentucky Early Mathematics Testing Program. He has been involved in curriculum alignment issues in mathematics for over ten years. He was a member of the national Content Expert/Employer Panel that developed the American Diploma Project benchmarks in mathematics. He was a member of the group that developed the postsecondary placement policy and college readiness standards in mathematics. He was a member of the Commissioner's Task Force that developed the core content standards in high school mathematics. He is one of several leaders of the Statewide Mathematics Placement Testing Project.

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